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Dynamical Transformations and Information Systems

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A description is given of dynamical transformations in the language of the theory of information systems. The dynamical transformations (morphisms) and sets of states (objects) form a Cartesian closed category, thus retaining the crucial consistency between structures and dynamics.

1. INTRODUCTION

In the majority of physical theories one talks about a "*physical system*" (a notion that is often quite opaque and misleading), methods of preparation of the system in definite "*states*," "*operations*" (which can be performed on the system and to which some devices called transmitters correspond), and *observables* (measurements on the system).

In the formalization of assertions about how a physical system behaves, a preliminary but important point to realize is that very much depends on the vocabulary which emerges from the matching of intuitive ideas with idealizations of experimental procedures. In Posiewnik (1985) we proposed a language that designed for reasoning about preparation procedures. Our concern was to analyze and formalize the patterns of thought that are used in each preliminary stage of a physical experiment. The character of our language is as much that of an empirical study as that of an intellectually creative one: it uses Scott's theory of domains for denotational semantics to describe the phenomenology of physical state preparation processes. We tried to develop a semantic analysis of the notion of physical state and investigate the properties of the set of states equipped with a physically meaningful topology.

The theory of domains, making some questions precise and interesting, could form part of a bridge between scientific practice (preparation,

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measurement, transmission, model construction, simulation) and the mathematically well-developed but often nonconstructive theories of continuously varying quantities.

In all fundamental physical theories there is a profound and intimate connection between the mathematical structure of the theories, their conceptual structure for physical description, and their basic ontology. It was Bohr's great idea that the *form* of a coherent communication has to be in harmony with its *content.*

Wheeler (1982) writes; "For the world of physics as for the alphanumeric printout of the computer, the yes, no character of what is going on may not be apparent but it is behind the scene."

Complying with that, we conjecture that similar structures (maybe on a very general level of abstraction—where we understand "level" in the sense of category theory) should be used in the description of physical processes and in the foundations of the mathematical theory of computation.

2. INFORMATION SYSTEMS

Here we give a brief outline of the theory of information systems. For details see Scott (1982) and Posiewnik (1985).

Definition (Scott, 1982).

(i) An *information system* is a structure

$$(D, \Delta, \mathrm{Con}, \vdash)$$

where D is a set, Δ is a distinguished member of D (the *least informative* member), Con is a set of finite subsets of D (the *consistent* sets), and $\vdash$ is a binary relation between members of Con and members of D (the *entailment relation*).

Concerning Con, the following axioms must be satisfied for all finite subsets $u, v, \subseteq D$:

1. $u \in \mathrm{Con}$ whenever $u \subseteq v \in \mathrm{Con}$.
2. $\{X\} \in \mathrm{Con}$ whenever $X \in D$.
3. $u \bigcup \{X\} \in \mathrm{Con}$ whenever $u \vdash X$.

Concerning $\vdash$, the following axioms must be satisfied for all $u, v \in \mathrm{Con}$ and all $X \in D$:

4. $u \vdash \Delta$.
5. $u \vdash X$ whenever $X \in u$.
6. If $v \vdash Y$, for all $Y \in u$, and $u \vdash X$, then $v \vdash X$.

In the description of the phenomenology of the preparation process we may think of the members of D as properties in the Jauch–Piron sense (Piron, 1976) which an individual physical system under study may have.

Δ is the trivial property engraved in each state of the system (e.g., the property that the system exists). If $u \in \mathrm{Con}$ is false, then the properties from u are never simultaneously actual.

The relation $\vdash$ is interpreted here in the sense of the semantic relation of implication.

(ii) The *states* (elements) of a physical system represented by an information system $A = (D_A, \Delta_A, \mathrm{Con}_A, \vdash_A)$ are those subsets x of D_A where (a) all finite subsets of x are in Con_A; and (b) whenever $u \subseteq x$ and $u \vdash_A X$, then $X \in x$.

We write $x \in |A|$ to mean x is a state of the system A. A state that is not included in any strictly larger state in the set $|A|$ is called a *pure* state.

(iii) Let A be an information system. The *topology* in the set $|A|$ is generated by a family of neighborhoods of the form

$$[u]_A = \{y \in |A|: \quad u \subseteq y\}$$

where $u \in \mathrm{Con}_A$. The neighborhoods of a state x are all those sets $[u]_A$ where $u \subseteq x$. Therefore a neighborhood of a state $x \in |A|$ generated by a set $u \in \mathrm{Con}_A$ is a set of all these states that differ from the state x by no more than u.

3. DYNAMICS

In this paper we would like to describe in the language and spirit of the theory of information systems the operations on states, i.e., changes of physical states due to external influences (transmitters) to which the system is subjected. We may treat a transmitter as a black box with one input channel and one output channel. A system prepared in some state enters the input channel and after transformation leaves the output channel. Because all change is basically qualitative, what we really observe in the laboratory is the change of the properties of our system. We always know at most only *some* properties of a state, and so an *approximation* to the state. We would like to give a theory of the transmission process that in some suitable sense preserves the spirit of approximation.

During the transmission process some actual properties may become potential, some potential ones may become actual, and some actual changes may be to other actual ones. We assume that the transmitter under study is a deterministic one in the Daniel-Gisin sense (Daniel, 1982), i.e., no property may become actual in a stochastic way and for one input state from the transmitter domain we have at most one output state.

The second simplifying assumption is that during the transmission the system and transmitter remain the same. So on the most primitive level we can represent the transmission process as a relation from the set Con_A to

Con_A. If f denotes the relation, then ufv means that for each input state with a consistent set u of actual properties, after the transmission process we obtain an output state possessing at least the consistent set v of actual properties.

What one can say about the relation f?

1. If we only know that the input state belongs to the domain of the transmitter under study, then the only thing we can say about the transmission process is that the same system leaves the output channel or that it exists after the operation.

In the language of information systems the above can be written

$$\Delta_A f \Delta_A$$

2. Suppose that for each preparation procedure of a state with consistent set u of actual properties, after transmission operation we always obtain a state for which at least properties v are actual ($v \in \text{Con}_A$) and then after perhaps some other observations we conclude that every output state has a consistent set v' of actual properties as well. Thus v and v' approximate the image of u and moreover there is an input state x ($x \supseteq u$) for which the output state has the properties v and v' simultaneously actual. Therefore, it is rather obvious that $v \cup v'$ belongs to Con_A and we can write the above assertions in the following way:

$$ufv \quad \text{and} \quad ufv' \qquad \text{always imply} \qquad uf(v \cup v')$$

3. For $u, u' \in \text{Con}_A$, $u' \vdash_A u$ means that whenever all the properties in u' are actual for a state of the system under study, then u are actual for the same state. So if the input properties are strengthened while the output ones are weakened, then the relation f must hold:

$$u' \vdash_A u, \quad ufv, \quad \text{and} \quad v \Vdash_A v' \qquad \text{always imply} \qquad u'fv'$$

But points 1–3 are exactly Scott (1982) conditions on an *approximable mapping*.

All this explains the following:

Assumption. Any deterministic transmitter may be represented as an approximable mapping.

Definition (Scott, 1982). If $f: A \to B$ is an approximable mapping between information systems, and if $x \in |A|$ is an element, then we define the *image* of x under f by the formula

$$f(x) = \{Y \in D_B \colon uf\{Y\} \text{ for some } u \subseteq x\}$$

or the equivalent formula

$$f(x) = \bigcup \{v \in \text{Con}_B \colon ufv \text{ for some } u \subseteq x\}$$

One can easily show that the image of a state in $|A|$ is a state in $|B|$.

The definition is a very natural one: the output state is simply a collection of all output properties for the same input state.

Proposition (Scott, 1982). Let $f, g: A \to B$ be two approximable mappings between two information systems. Then:

1. $f = g$ iff $f(x) = g(x)$ for all $x \in |A|$.
2. $x \subseteq y$ in $|A|$ always implies $f(x) \subseteq f(y)$ in $|B|$.

Moreover, approximable mappings correspond exactly to continuous mappings between topological spaces $|A|$ and $|B|$. The totality of information systems and approximable mappings form a (Cartesian closed) category (Scott, 1982).

The above considerations assure us that we are in agreement with the doctrines of the Eilenberg–MacLane category theory program, which states that any species of mathematical structure should be represented by a *category*, whose *objects* "are of that structure" and whose *morphisms* "preserve" it. In our case *objects* represent the sets of states of physical systems and *morphisms* the dynamical transformations, and the crucial consistency between structure and dynamics is retained.

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REFERENCES

Daniel, W. (1982). *Helvetica Physica Acta*, **55**, 330.
Piron, C. (1976). *Foundations of Quantum Physics*, Reading, Massachusetts.
Posiewnik, A. (1985). *International Journal of Theoretical Physics*, **24**, 135.
Scott, D. (1982). Domains for denotational semantics, preprint.
Wheeler, J. A. (1982). *International Journal of Theoretical Physics*, **21**, (6/7).